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Discrete Math

1.1 - Introduction to Sets

A *set* is a collection of things. The things in a set are called *elements* of the set.

Ex (a) $\{1, 2, 3, 4\}$
(b) $\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$
(c) $\mathbb{R} = \{x \mid -\infty < x < \infty\}$

There are some adjectives we can attach to "set". A set is called *infinite* if it has infinitely many elements, like $\mathbb{Z}$ and $\mathbb{R}$. A set with finitely many elements is called *finite*. There are two types of infinite sets: *countable* and *uncountable*.

Ex: $\mathbb{Z}$ is countable while $\mathbb{R}$ is not.

We will talk more about this distinction later, but it essentially means that you can list the elements when it's countable.

We say two sets are *equal* if they have the same elements.

Ex: ⓐ $\{6,12,18,24\} = \{18,12,24,6\}$

ⓑ $\{6,12,18,24\} \neq \{6,12,18\}$

ⓒ $\{0,1,-1,2,-2,3,-3,\cdots\} = \{\cdots,-3,-2,-1,0,1,2,3,\cdots\}$

To denote that an element belongs to a set we use the symbol "$\in$" and "$\notin$" when an element does not belong to a set.

Ex: $2 \in \mathbb{Z}$ but $\frac{1}{2} \notin \mathbb{Z}$

Some Special Sets

Natural #'s $\quad \mathbb{N} = \{1,2,3,\cdots\}$

Integers $\quad \mathbb{Z} = \{\cdots,-2,-1,0,1,2,\cdots\}$

Rational #'s $\quad \mathbb{Q} = \{\frac{a}{b} \mid a,b \in \mathbb{Z},\ b \neq 0\}$

Real #'s $\quad \mathbb{R} = (-\infty,\infty)$

Complex #'s $\quad \mathbb{C} = \{a+ib \mid a,b \in \mathbb{R}\}$

Empty Set $\quad \emptyset = \{\}$

The empty set is a very important set, as we will see as time goes on.

We will also be interested in the size or <u>cardinality</u> of a set. For a set A, denote its cardinality by $|A|$. For finite sets, the cardinality of the set is the number of elements in the set. We'll discuss infinite sets later.

<u>Ex</u>: ⓐ $|\{2,4,6,8,10\}| = 5$

ⓑ $|\emptyset| = 0$

ⓒ $|\{(1,0),(0,1),(1,1),(0,0)\}| = 4$

ⓓ $|\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}| = 3$

As we have seen some examples of already, we can specify sets by using the notation

$$X = \{\text{expression} \mid \text{rule}\}$$

We read these in the following way:

Ex: $A = \{\frac{1}{n} \mid n \in \mathbb{Z}\}$ is read as "A is the set of elements of the form $\frac{1}{n}$ such that n is an integer."

This notation is called <u>set builder notation.</u>

Ex: Write the following sets in set builder notation:

(a) $\{2, 3, 5, 7, 11, 13, 17, 19, \ldots\}$
(b) $\{1, 4, 9, 16, 25, \ldots\}$
(c) All real numbers with absolute value less than π.

Sol: (a) $\{p \in \mathbb{N} \mid p \text{ is prime}\}$
(b) $\{n^2 \mid n \in \mathbb{N}\}$
(c) $\{x \in \mathbb{R} \mid |x| < \pi\}$

1.2 - Cartesian Product

Given two sets A and B we can form a new set, denoted by $A \times B$, by taking the <u>Cartesian product</u>. The product is defined as a set of <u>ordered pairs</u>.

$$A \times B := \{(a, b) \mid a \in A, b \in B\}$$

⑤

Since we use ordered pairs, in general $(a,b) \neq (b,a)$ so $\boxed{A \times B \neq B \times A}$

(though there is a sense of sameness)

Ex: Write out the set $A \times B$ where $A = \{1,2,3,4\}$ and $B = \{a,c\}$.

Sol:

$$A \times B = \{(1,a),(1,c),(2,a),(2,c),(3,a),(3,c),(4,a),(4,c)\}$$

Fact: For finite sets A and B

$$|A \times B| = |A| \cdot |B|$$

(this is also true for infinite sets, but one needs to understand what $|A|$ means there and how to multiply infinite cardinals)

We don't have to stop at taking the product of two sets. For example:

$$A \times (B \times C) = \{(a,(b,c)) \mid a \in A, b \in B, c \in C\}$$

Similar to the reasons above

$$\boxed{A \times (B \times C) \neq (A \times B) \times C}$$